N-element Array : Uniform Amplitude and spacing

Uniform Array:

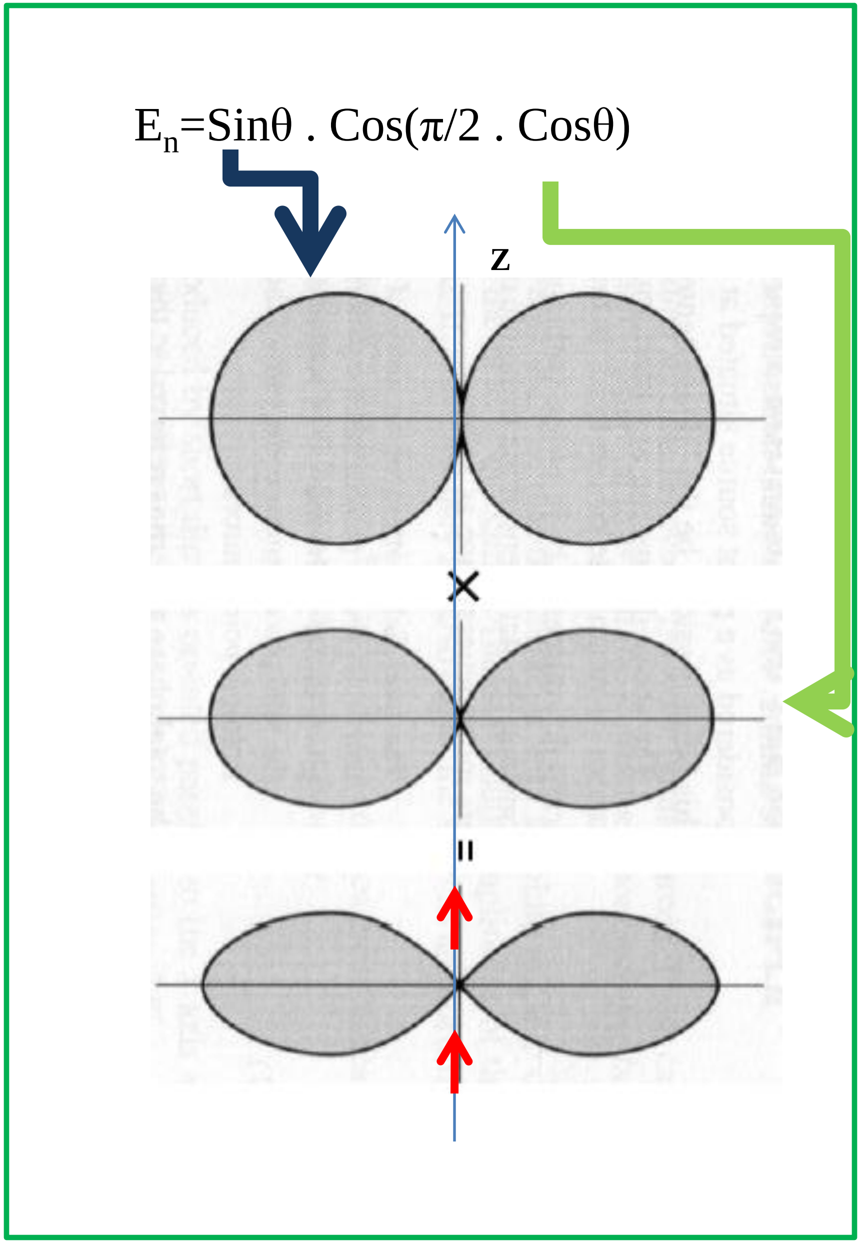
- Identical elements with identical amplitudes
- Progressive phase shift

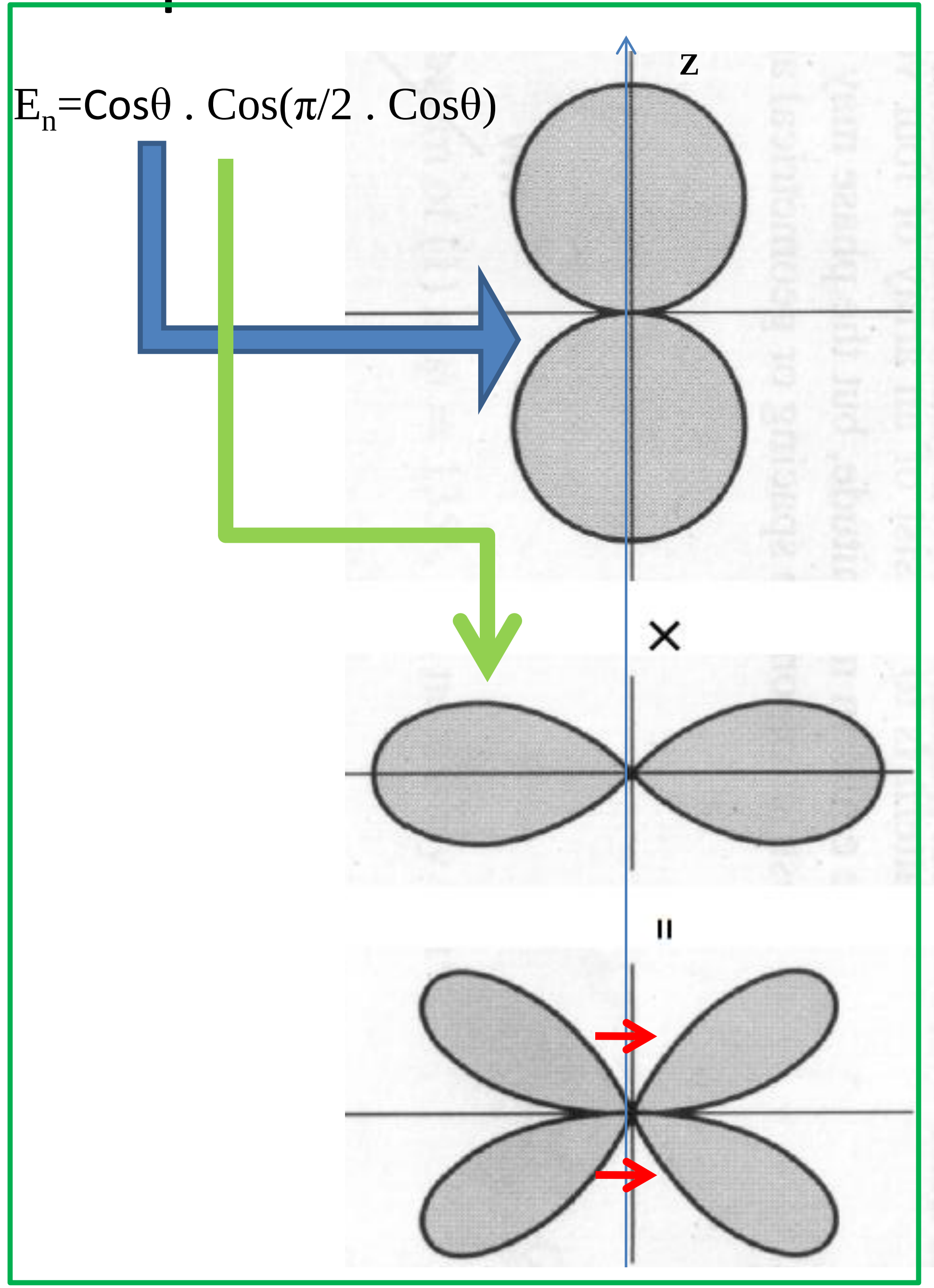
Q:Define pattern multiplication rule:

Field pattern of an array of non-isotropic identical elements (which has identical magnitude and progressive phase) formed from product of pattern of single element X pattern of array of isotropic point sources has same location and progressive phases as of non isotropic point sources.

• Examples of Pattern Multiplication

Array of two infinitesimal elements λ/2 spacing





Why antenna Array

1-Usually gain of single element is low, thus array is used for **increasing gain** for long distance communication

____

If λ/2 dipole is reference

i.e. its Gain considered to be=0dB

("note that λ/2 dipole has D=2dB")

then

2 element array increase gain by 3dB(double gain 2 time)

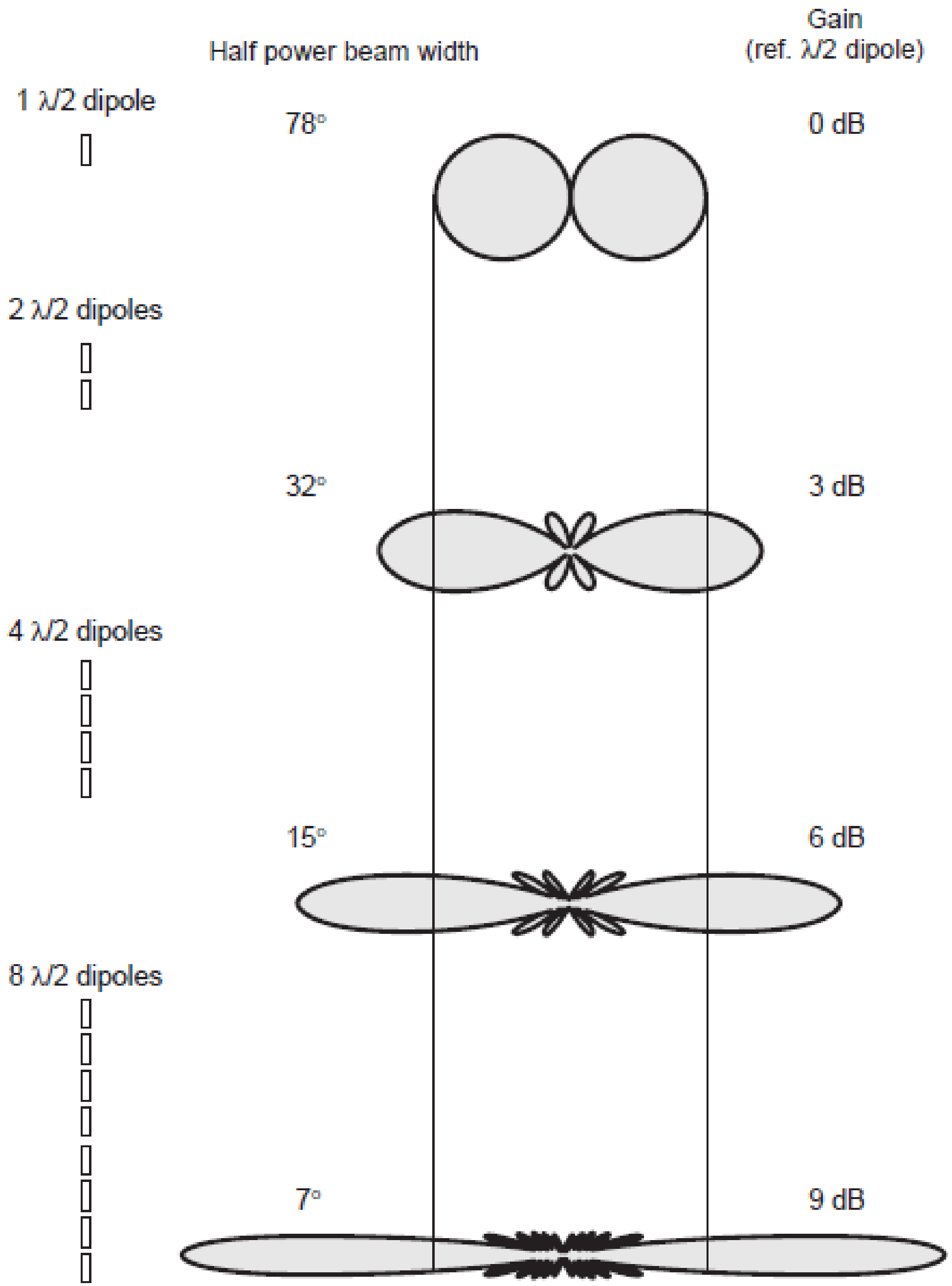
4 element array increase gain by 6dB(double gain 4 time)

8 element array could increase gain by 9dB(double gain 8time)

2-Beam steering

by changing progressive phase

3-Nulling interference directions



Broad side array β=0

- ***N-ELEMENT LINEAR ARRAY: Uniform Amplitude* and Spacing**

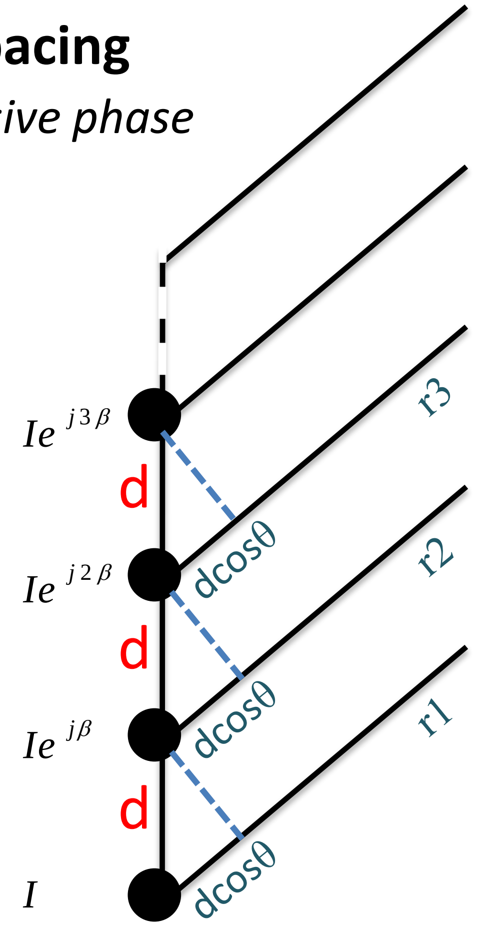
uniform array has: Identical elements-Identical magnitude-Progressive phase

Also uniform spacing

- $Etot = E1 + E2 + E3 + \ldots + EN$

- $Etot = A\,[I_1 e^{jkr1} + I_2 e^{jkr2} + I_3 e^{jkr3} + \ldots\ldots + I_N e^{jkrN}]$
- Where $A = (j\eta kL/4\pi r)\sin\theta$ for infinitesimal dipole
- $I_2 = I_1 e^{j\beta}$
- $r_2 = r_1 - d\cos\theta$

- $Etot = A\, I_1 e^{jkr1}[1 + e^{j\beta} e^{jkd\cos\theta} + e^{j2\beta} e^{j2kd\cos\theta} + e^{j3\beta} e^{j3kd\cos\theta} + \ldots]$



$$\mathbf{AF = 1 + e^{j(kd\cos\theta+\beta)} + e^{j2(kd\cos\theta+\beta)} + e^{j3(kd\cos\theta+\beta)} + \ldots + e^{j(N-1)(kd\cos\theta+\beta)}}$$

$$\mathbf{AF = 1 + e^{j\psi} + e^{j2\psi} + e^{j3\psi} + \ldots + e^{j(N-1)\psi}}$$

Where, $\psi = kd\cos\theta + \beta$

- **AF = 1+ $e^{j\psi}$ +$e^{j2\psi}$ +$e^{j3\psi}$ +…..+$e^{j(N-1)\psi}$** *(1)*
- **AF. $e^{j\psi}$ = $e^{j\psi}$ +$e^{j2\psi}$ +$e^{j3\psi}$ +…..+$e^{j(N-1)\psi}$+ $e^{jN\psi}$** *(2)*
- **Subtract (1) from (2)** **AF$(e^{j\psi} - 1) = (-1+ e^{jN\psi})$**

$$\mathrm{AF} = \left[\frac{e^{jN\psi} - 1}{e^{j\psi} - 1}\right] = e^{j[(N-1)/2]\psi}\left[\frac{e^{j(N/2)\psi} - e^{-j(N/2)\psi}}{e^{j(1/2)\psi} - e^{-j(1/2)\psi}}\right]$$

$$= e^{j[(N-1)/2]\psi}\left[\frac{\sin\left(\frac{N}{2}\psi\right)}{\sin\left(\frac{1}{2}\psi\right)}\right]$$

Max occurred at $\mathrm{AF} = \frac{0}{0}$ **which occurred at $\psi = \pm m\pi$ (for m=0,1,2,….)**

To get Max value differentiate num and denum w.r.t. ψ *(and substitute ψ=0)*

AF_{MAX}= N hence

$$(\mathrm{AF})_n = \frac{1}{N}\left[\frac{\sin\left(\frac{N}{2}\psi\right)}{\sin\left(\frac{1}{2}\psi\right)}\right]$$

Observations:

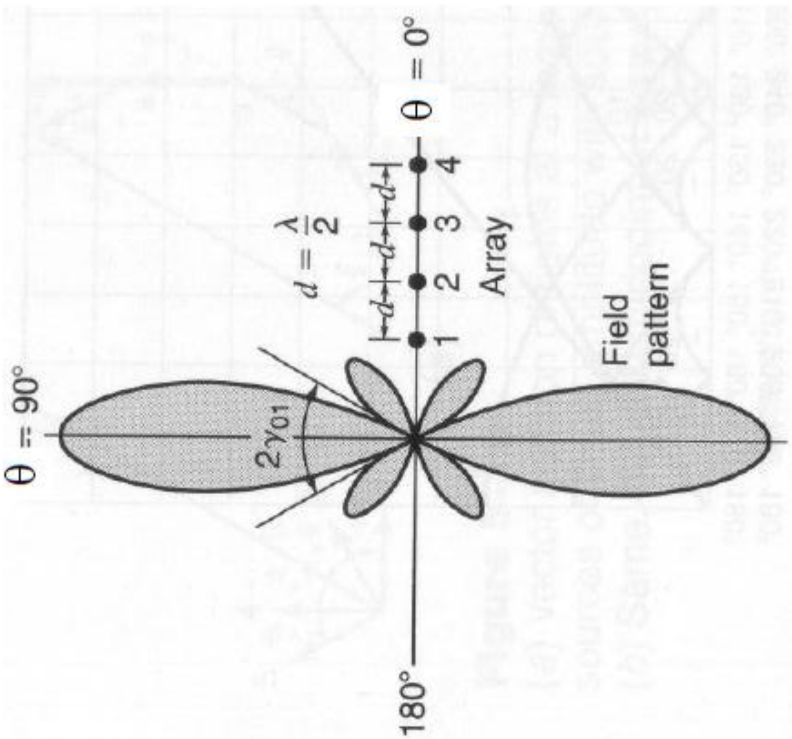
(1) Main lobe is in the direction so that

$\psi = k\, d \cos\theta + \beta = 0$

(2) The main lobe narrows as N increases.

- Max occurred at $\psi = 0 = k.d.\cos\theta + \beta$ (for AF pattern)

Broad Side Array

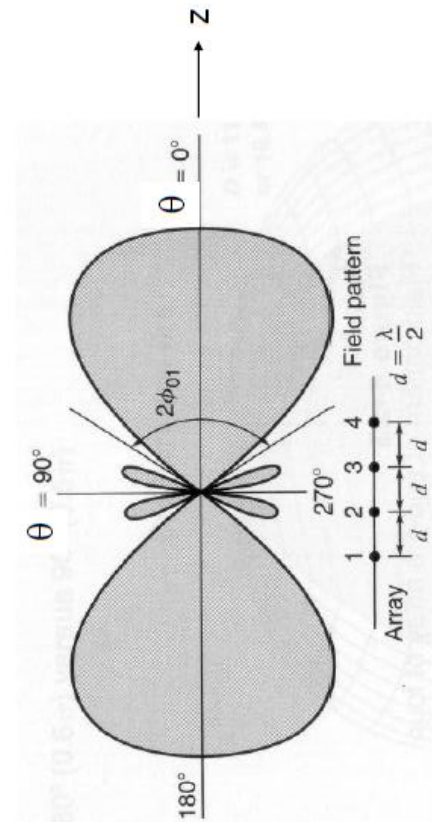


Since it is desired to have the first maximum directed toward $\theta_0 = 90^\circ$, then

Setting For broad side AF pattern

$$\psi = kd\cos\theta + \beta|_{\theta=90^\circ} = \beta = 0$$

End Fire Array



Setting For End Fire AF pattern

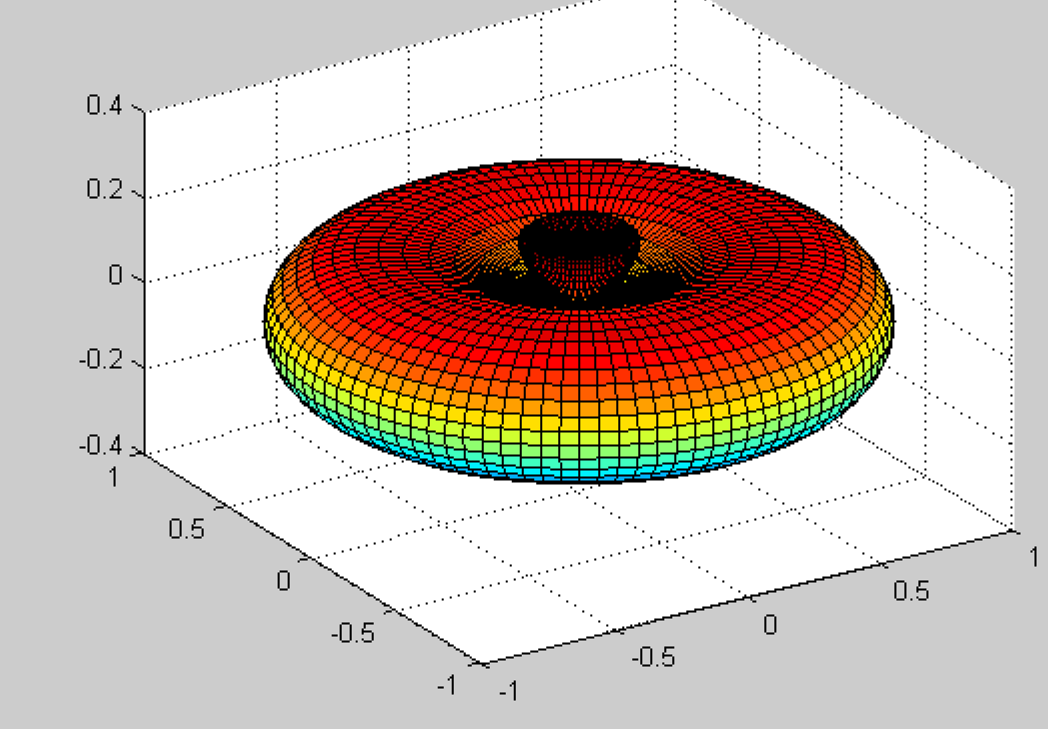
To direct the first maximum toward $\theta_0 = 0^\circ$,

$$\psi = kd\cos\theta + \beta|_{\theta=0^\circ} = kd + \beta = 0 \Rightarrow \beta = -kd$$

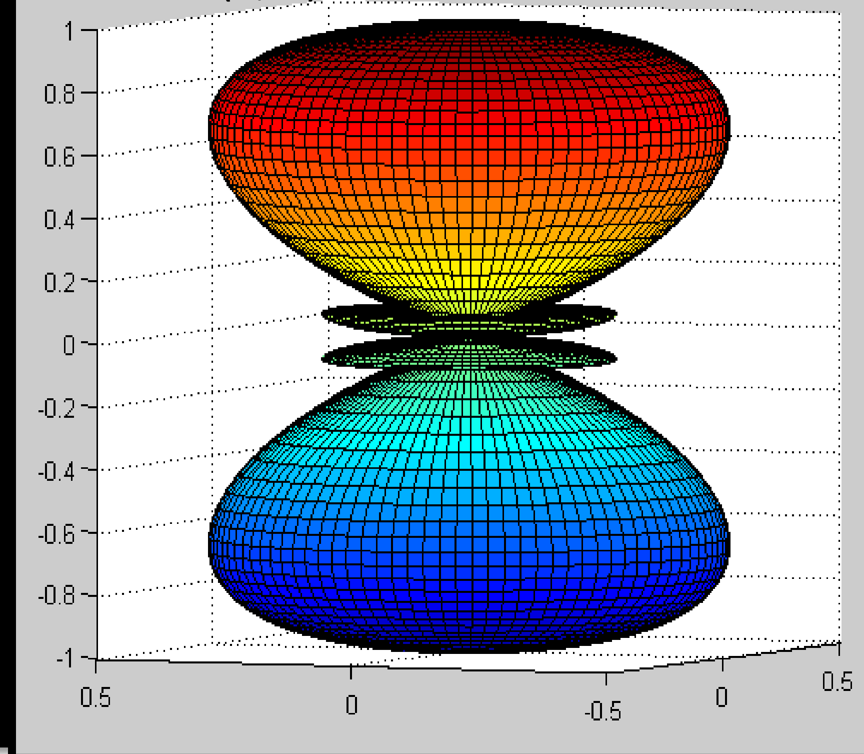
If the first maximum is desired toward $\theta_0 = 180^\circ$, then

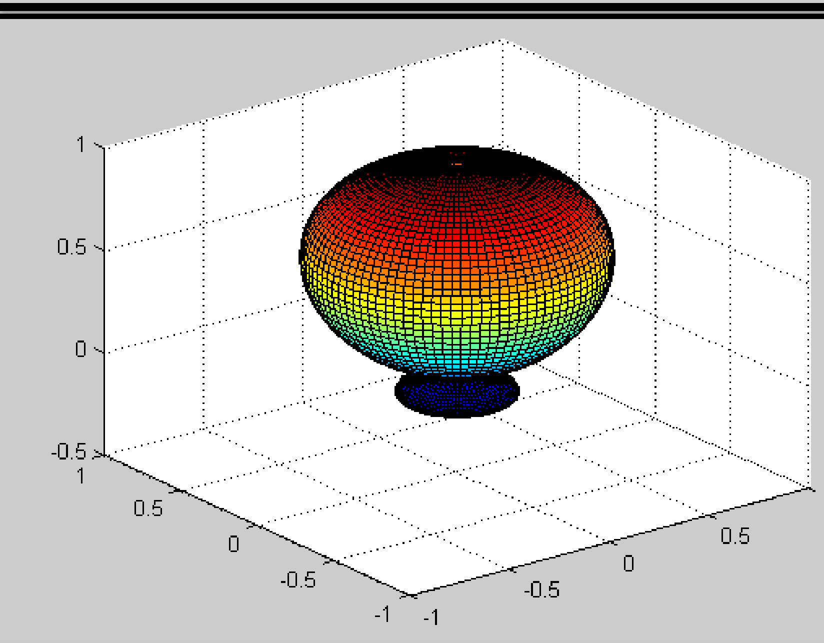
$$\psi = kd\cos\theta + \beta|_{\theta=180^\circ} = -kd + \beta = 0 \Rightarrow \beta = kd$$

broadside(4,.5)



>> endfire(4,.5)






```
>> endfire(4,.25)
%endfire(N,d)
%note for d<λ/4 only one main lobe directed at θ=0
```

It is required to study $(AF)_n$

$$(AF)_n = \frac{1}{N}\left[\frac{\sin\left(\frac{N}{2}\psi\right)}{\sin\left(\frac{1}{2}\psi\right)}\right]$$

- **Nulls**

$$N\frac{\psi}{2} = \pm n\pi \quad , \quad m = 1,2,3,.. \neq 0,\ N,2N,....$$

- **Maximum**

$$\frac{\psi}{2} = \pm m\pi \quad , \quad m = 0,1,2,.... \ (\ 0 \text{ for main lobe })$$

Grating lobe condition (at m=1,2,3,...)

- **3-dB point**

$$N\frac{\psi}{2} = 1.39$$

- **Secondary Maximum for minor lobes**

$$N\frac{\psi}{2} = \frac{2s+1}{2}\pi \quad , \quad s = 1,\ 2,\ 3,.....$$

Maximum of first minor lobe occurred at $N\psi/2=3\pi/2$

$$(AF)_n = \frac{1}{N}\left[\frac{\sin\left(\frac{N}{2}\psi\right)}{\sin\left(\frac{1}{2}\psi\right)}\right]$$

- **NULLS**
- Nulls occurred at *sin(Nψ/2)*=0
- Kdcosθ+ β=±2nπ/N   where n=1,2,3 (again n≠ 0 or N or 2N…….this make $(AF)_n$=0/0 which is max condition)

$$\theta_n = \cos^{-1}\left[\frac{\lambda}{2\pi d}\left(-\beta \pm \frac{2n}{N}\pi\right)\right]$$

- **Broadside Array (sources in phase β=0)**

$$\theta_n = \cos^{-1}\left(\pm\frac{n}{N}\frac{\lambda}{d}\right)$$
$$n = 1, 2, 3, \ldots$$
$$n \neq N, 2N, 3N, \ldots$$

End fire Array (β=-kd)

$$\theta_n = \cos^{-1}\left(1 - \frac{n\lambda}{Nd}\right)$$
$$n = 1, 2, 3, \ldots$$
$$n \neq N, 2N, 3N, \ldots$$

1- because $\cos^{-1}$(less than 1)

$$(\text{AF})_n = \frac{1}{N}\left[\frac{\sin\left(\frac{N}{2}\psi\right)}{\sin\left(\frac{1}{2}\psi\right)}\right] \simeq \left[\frac{\sin\left(\frac{N}{2}\psi\right)}{\frac{N}{2}\psi}\right]$$

- MAXIMUM
- Maximum occurred at $\psi/2 = \pm m\pi$ (for $(AF)_n$=0/0)
- Kdcosθ+ β=±2mπ where m=0,1,2,3

$$\theta_m = \cos^{-1}\left[\frac{\lambda}{2\pi d}(-\beta \pm 2m\pi)\right]$$

- For sin(Nx)/Nsin(x)maximum occurred at x=0 or $\psi/2=0$ *i.e* m=0
-
- **Broa**

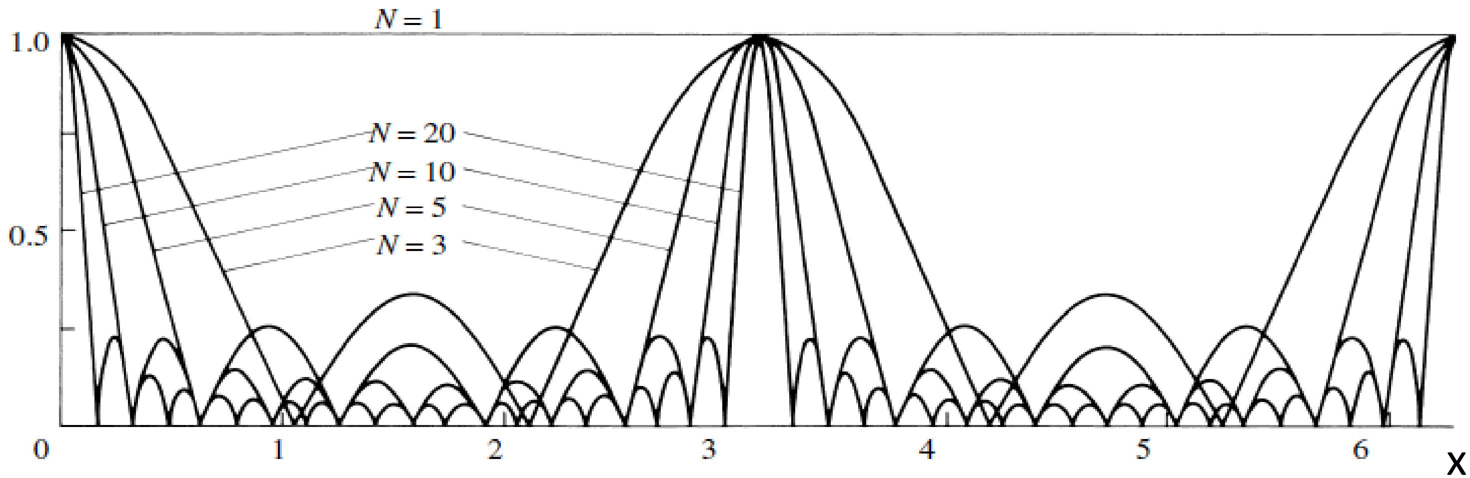


Figure II.1 Curves of |sin(Nx)/N sin(x)| function.

Grating lobe condition

- Grating Lobe is lobe with Maxima(as of major) in other direction (un required direction).

Max condition array factor was(0/0 condition) at $\psi/2=0$ **i.e** $Kd\cos\theta_g+\beta=\pm 2m\pi$ **,**

- **for broad side** $\beta=0$

$\theta_m=\cos^{-1}(m\lambda/d)$ there is no grating lobe as long as **$d_{max}<\lambda$**

$\cos^{-1}(m\lambda/d)$ exist only at m=0 when $d_{max}<\lambda$

if d realize $m\lambda/d<1$ or $d>m\lambda$ there is grating lobe.

- **For end fire**, $\beta=-kd$

for $\theta_m=\cos^{-1}(1-n\lambda/d)$ there is no grating lobe as long as **$d_{max}<\lambda/2$**

- **$\cos^{-1}(1-n\lambda/d)$ exist only at m=0 when $d_{max}<\lambda/2$**

$$(\mathrm{AF})_n = \frac{1}{N}\left[\frac{\sin\left(\frac{N}{2}\psi\right)}{\sin\left(\frac{1}{2}\psi\right)}\right] \simeq \left[\frac{\sin\left(\frac{N}{2}\psi\right)}{\frac{N}{2}\psi}\right]$$

- 3-dB point for AF
- Use Approximation sin(x)/x because it does not depend on N

Using try and error 3dB occurred at sin(x)/x=.707 i.e. x=1.39

*because it is field pattern (sin(1.93*180/π)/1.39=.7076*

x	$\sin(x)/x$
1.3	0.74120
1.4	0.70389

$$\frac{N}{2}\psi = \frac{N}{2}(kd\cos\theta + \beta)|_{\theta=\theta_h} = \pm 1.391$$

$$\Rightarrow \theta_h = \cos^{-1}\left[\frac{\lambda}{2\pi d}\left(-\beta \pm \frac{2.782}{N}\right)\right]$$

- **Broadside Array (sources in phase β=0)**

End fire Array (β=-kd)

HALF-POWER POINTS $\theta_h \simeq \cos^{-1}\left(\pm\frac{1.391\lambda}{\pi N d}\right)$

$$\theta_h \simeq \cos^{-1}\left(1 - \frac{1.391\lambda}{\pi d N}\right)$$

$$(\mathrm{AF})_n = \frac{1}{N}\left[\frac{\sin\left(\frac{N}{2}\psi\right)}{\sin\left(\frac{1}{2}\psi\right)}\right] \simeq \left[\frac{\sin\left(\frac{N}{2}\psi\right)}{\frac{N}{2}\psi}\right]$$

- # Secondary Maximafor AF

Secondary maxima occurred at numerator is maxima

$$\sin\left(\frac{N}{2}\psi\right) = \sin\left[\frac{N}{2}(kd\cos\theta + \beta)\right]\Big|_{\theta=\theta_s} \simeq \pm 1 \Rightarrow \frac{N}{2}(kd\cos\theta + \beta)|_{\theta=\theta_s}$$

$$\simeq \pm\left(\frac{2s+1}{2}\right)\pi \Rightarrow \theta_s \simeq \cos^{-1}\left\{\frac{\lambda}{2\pi d}\left[-\beta \pm \left(\frac{2s+1}{N}\right)\pi\right]\right\}$$

$$s = 1, 2, 3, \ldots$$

- Maximum of first minor lobe occurred at Nψ/2=3π/2 (i.e. s=1)

$$(AF)_n = \frac{\sin(\frac{N\psi}{2})}{\frac{N\psi}{2}} = \frac{1}{3\pi/2} = .212 = -13\,dB$$

20log() as it is amplitude

- **Broadside Array (sources in phase β=0)**

MINOR LOBE MAXIMA

$$\theta_s \simeq \cos^{-1}\left[\pm\frac{\lambda}{2d}\left(\frac{2s+1}{N}\right)\right]$$

$$s = 1, 2, 3, \ldots$$

End fire Array (β=-kd)

$$\theta_s \simeq \cos^{-1}\left[1 - \frac{(2s+1)\lambda}{2Nd}\right]$$

$$s = 1, 2, 3, \ldots$$

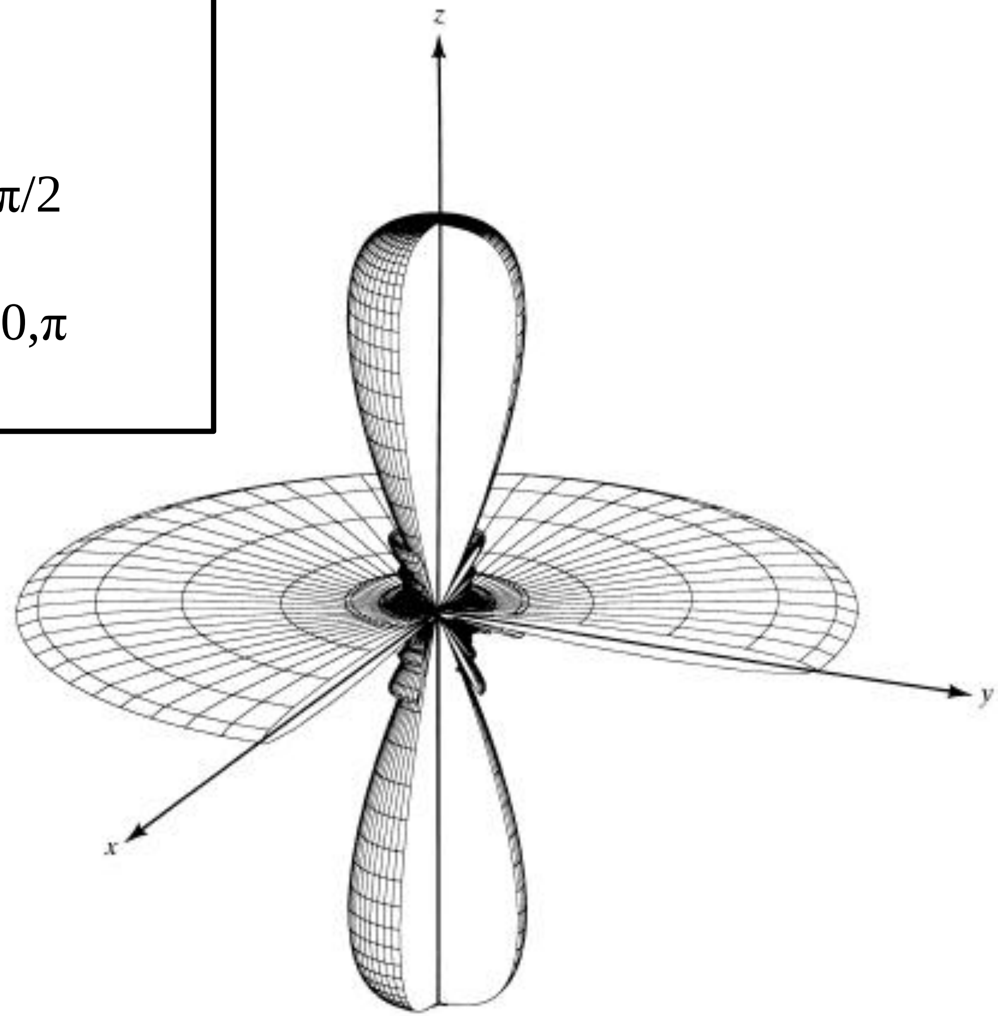
For Broadside array β=0, d=λ

$\theta_m = \cos^{-1}(\pm m\,\lambda/d) = \cos^{-1}(\pm m)$

exist at

m=0(main lobe) $\theta_m = \pi/2$

m=1 $\theta_m = 0, \pi$



(b) Broadside/end-fire ($\beta = 0$, $d = \lambda$)

Figure 6.6 Three-dimensional amplitude patterns for broadside, and broadside/end-fire arrays ($N = 10$).

Broadside Array

For d=λ maxima at 0,90,180

d=λ/4 maxima at 90

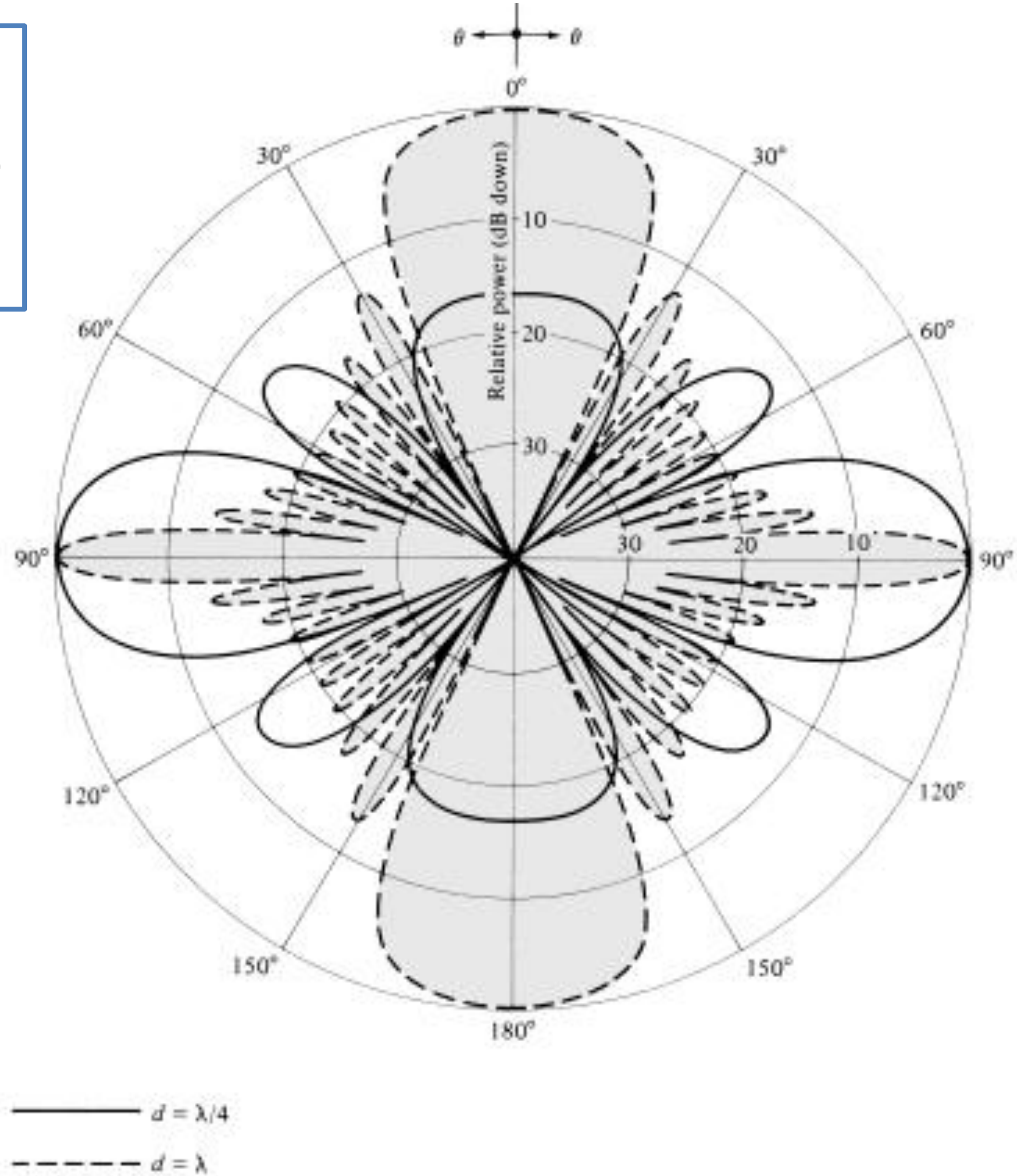


Figure 6.7 Array factor patterns of a 10-element uniform amplitude broadside array ($N = 10, \beta = 0$).

	Broad side	End fire
NULLS	$\theta_n = \cos^{-1}\left(\pm\frac{n}{N}\frac{\lambda}{d}\right)$ $n = 1, 2, 3, \ldots$ $n \neq N, 2N, 3N, \ldots$	$\theta_n = \cos^{-1}\left(1 - \frac{n\lambda}{Nd}\right)$ $n = 1, 2, 3, \ldots$ $n \neq N, 2N, 3N, \ldots$
MAXIMA	$\theta_m = \cos^{-1}\left(\pm\frac{m\lambda}{d}\right)$ $m = 0, 1, 2, \ldots$	$\theta_m = \cos^{-1}\left(1 - \frac{m\lambda}{d}\right)$ $m = 0, 1, 2, \ldots$
HALF-POWER POINTS	$\theta_h \simeq \cos^{-1}\left(\pm\frac{1.391\lambda}{\pi Nd}\right)$ $\pi d/\lambda \ll 1$	$\theta_h \simeq \cos^{-1}\left(1 - \frac{1.391\lambda}{\pi dN}\right)$ $\pi d/\lambda \ll 1$
MINOR LOBE MAXIMA	$\theta_s \simeq \cos^{-1}\left[\pm\frac{\lambda}{2d}\left(\frac{2s+1}{N}\right)\right]$ $s = 1, 2, 3, \ldots$	$\theta_s \simeq \cos^{-1}\left[1 - \frac{(2s+1)\lambda}{2Nd}\right]$ $s = 1, 2, 3, \ldots$